FITXA DE POLINOMIS (1)

Operacions, Ruffini, factor comú, identitats notables

1. **Efectua i simplifica**

    a) $(2x^2 - 3x + 1)(2x - 1) - (4x^3 - 8x^2 + 1)$
    b) $(x^2 - 3)(2x + 1) - (2x^3 + x^2 - 3x)$
    c) $(3x^2 - 1)(2x^2 + 5x) - (6x^4 + 15x^3 + 2x^2 - 1)$
    d) $(x^2 - 2x + 3)(x - 2) - (x^3 - 4x^2 + 3)$
    e) $(-2x^2 + x - 1)(3x + 1) - (-6x^3 + x^2 - 5x)$

2. **Calcula el quocient i el residu d'aquesta divisió:**

    a) $\left(-2x^4 + 3x^3 - 2x + 3\right) : \left(x^2 - 2x + 2\right)$
    b) $\left(2x^4 - 7x^3 + 3x^2 - 1\right) : \left(x^2 + 2\right)$
    c) $\left(4x^3 - 2x^2 + 5x + 3\right) : \left(x^2 - 2\right)$
    d) $(2x^5 - 3x^4 + 2x^2 - x + 1) : (x^3 - 2x + 1)$
    e) $\left(5x^4 + x^3 - 2x^2 + 1\right) : \left(x^2 - 3x + 1\right)$

3. **Calcula, utilitzant la regla de Ruffini, el quocient i el residu d'aquesta divisió:**

    a) $\left(5x^4 - x^3 + 2x - 1\right) : \left(x + 2\right)$
    b) $\left(2x^5 - 3x^3 + 2x - 1\right) : \left(x + 2\right)$
    c) $\left(x^4 - 3x^3 + 2x^2 + 5\right) : \left(x + 1\right)$
    d) $\left(-x^4 + 2x^2 - x + 2\right) : \left(x + 2\right)$
    e) $\left(-3x^4 + 6x^2 + x - 2\right) : \left(x - 1\right)$

4. **Utilitza les identitats notables per a calcular:**

    a) $(5x + 2)^2$

    b) $(3x - 7)^2$

    c) $\left(x - \dfrac{1}{2}\right)\left(x + \dfrac{1}{2}\right)$

    d) $(6x - 2)(6x + 2)$
    e) $(8x + 2)^2$
    f) $(6x - 3)^2$
    g) $(5x - 4)(5x + 4)$
    h) $(7x + 8)^2$
    i) $(5x - 1)^2$

5. **Calcula i simplifica:**

a) $(x+1)^2(3x^2+2)-2(x^4-x^3+2x^2-1)$

b) $(2x-3)^2-(2x^2+4x+1)(x-2)$

c) $\left(\dfrac{2}{3}x-1\right)(3x+6)+(x+1)(x-1)-(x+2)^2$

d) $(x-1)(x^2+x)^2-(x^5-5x^4+x^3-x^2)$

e) $(x^2-2x+3)(2x+1)-(4x-1)^2$

6. **Extreu factor comú en cadascun d'aquests polinomis:**

a) $5x^5-3x^3+6x$

b) $12x^5-8x^3+4x^2$

c) $x^7-3x^6+9x^5$

d) $12x^3-6x$

7. **Utilitza els productes notables per a descompondre en factors els polinomis següents:**

a) $25x^2+10x+1$

b) $9x^2-16$

c) $16x^2-40x+25$

d) $9x^2-64$

e) $9x^2-24x+16$

f) $64x^2-25$

8. **Treu factor comú i utilitza els productes notables per a factoritzar aquests polinomis:**

a) $4x^4-4x^3+x^2$

b) $2x^3-18x$

c) $x^6-6x^5+9x^4$

d) $12x^3-75x$

e) $9x^5+6x^4+x^3$

f) $18x^3-2x$

SOLUCIONS

Exercici 1

a) $(2x^2 - 3x + 1)(2x - 1) - (4x^3 - 8x^2 + 1) = (4x^3 - 2x^2 - 6x^2 + 3x + 2x - 1) - (4x^3 - 8x^2 + 1) =$

$= 4x^3 - 8x^2 + 5x - 1 - 4x^3 + 8x^2 - 1 = 5x - 2$

b) $(x^2 - 3)(2x + 1) - (2x^3 + x^2 - 3x) = 2x^3 + x^2 - 6x - 3 - 2x^3 - x^2 + 3x = -3x - 3$

c) $(3x^2 - 1)(2x^2 + 5x) - (6x^4 + 15x^3 + 2x^2 - 1) = 6x^4 + 15x^3 - 2x^2 - 5x - 6x^4 - 15x^3 - 2x^2 + 1 =$

$= -4x^2 - 5x + 1$

d) $(x^2 - 2x + 3)(x - 2) - (x^3 - 4x^2 + 3) = (x^3 - 2x^2 - 2x^2 + 4x + 3x - 6) - (x^3 - 4x^2 + 3) =$

$= x^3 - 4x^2 + 7x - 6 - x^3 + 4x^2 - 3 = 7x - 9$

e) $(-2x^2 + x - 1)(3x + 1) - (-6x^3 + x^2 - 5x) = -6x^3 - 2x^2 + 3x^2 + x - 3x - 1 + 6x^3 - x^2 + 5x =$

$= 3x - 1$

Exercici 2

a)

```
 − 2x⁴ + 3x³        − 2x + 3   │ x² − 2x + 2
   2x⁴ − 4x³  + 4x²            │ − 2x² − x + 2
 ──────────────────────
     −  x³  + 4x² − 2x + 3
        x³  − 2x² + 2x
    ──────────────────
             2x²        + 3
           − 2x² + 4x − 4
        ──────────────────
                  4x − 1
```

Quocient = $-2x^2 - x$ 2

Residu = $4x - 1$

b)

```
   2x⁴ + 7x³ +  3x²        − 1   │ x² + 2
  −2x⁴        − 4x²              │ 2x² − 7x − 1
 ──────────────────────
      − 7x³ −   x²        − 1
        7x³         + 14x
      ──────────────────
             −  x² + 14x − 1
                x²        + 2
           ──────────────────
                   14x + 1
```

Quocient = $2x^2 - 7x - 1$

Residu = $14x + 1$

c)

```
    4x³ − 2x² +   5x  + 3   │ x² − 2
  − 4x³         +   8x      │ 4x − 2
 ──────────────────────
        − 2x² + 13x  + 3
          2x²        − 4
       ──────────────────
              13x  − 1
```

Quocient = $4x - 2$
Residu = $13x - 1$

d)

$$
\begin{array}{l}
2x^5 - 3x^4 + 2x^2 - x + 1 \\
-2x^5 + 4x^3 - 2x^2 \\
\hline
-3x^4 + 4x^3 - x + 1 \\
3x^4 - 6x^2 + 3x \\
\hline
4x^3 - 6x^2 + 2x + 1 \\
-4x^3 + 8x - 4 \\
\hline
-6x^2 + 10x - 3
\end{array}
\quad
\begin{array}{|l}
x^3 - 2x + 1 \\
\hline
2x^2 - 3x + 4
\end{array}
$$

Quocient = $2x^2 - 3x + 4$
Residu = $-6x^2 + 10x - 3$

e)

$$
\begin{array}{l}
5x^4 + x^3 - 2x^2 + 1 \\
-5x^4 + 15x^3 - 5x^2 \\
\hline
16x^3 - 7x^2 + 1 \\
-16x^3 + 48x^2 - 16x \\
\hline
41x^2 - 16x + 1 \\
-41x^2 + 123x - 41 \\
\hline
107x - 40
\end{array}
\quad
\begin{array}{|l}
x^2 - 3x + 1 \\
\hline
5x^2 + 16x + 41
\end{array}
$$

Quocient = $5x^2 + 16x + 41$
Residu = $107x - 40$

Exercici 3

a)

$$
\begin{array}{r|rrrrr}
 & 5 & -1 & 0 & 2 & -1 \\
-2 & & -10 & 22 & -44 & 84 \\
\hline
 & 5 & -11 & 22 & -42 & 83
\end{array}
$$

Quocient = $5x^3 - 11x^2 + 22x - 42$
Residu = 83

b)

$$
\begin{array}{r|rrrrrr}
 & 2 & 0 & -3 & 0 & 2 & -1 \\
-2 & & -4 & 8 & -10 & 20 & -44 \\
\hline
 & 2 & -4 & 5 & -10 & 22 & -45 \\
\end{array}
$$

Quocient = $2x^4 - 4x^3 + 5x^2 - 10x + 22$
Residu = -45

c)

$$
\begin{array}{r|rrrrr}
 & 1 & -3 & 2 & 0 & 5 \\
-1 & & -1 & 4 & -6 & 6 \\
\hline
 & 1 & -4 & 6 & -6 & 11 \\
\end{array}
$$

Quocient = $x^3 - 4x^2 + 6x - 6$
Residu = 11

d)

$$
\begin{array}{r|rrrrr}
 & -1 & 0 & 2 & -1 & 2 \\
-2 & & 2 & -4 & 4 & -6 \\
\hline
 & -1 & 2 & -2 & 3 & -4 \\
\end{array}
$$

Quocient = $-x^3 + 2x^2 - 2x + 3$
Residu = -4

e)

$$
\begin{array}{r|rrrrr}
 & -3 & 0 & 6 & 1 & -2 \\
1 & & -3 & -3 & 3 & 4 \\
\hline
 & -3 & -3 & 3 & 4 & 2 \\
\end{array}
$$

Quocient = $-3x^3 - 3x^2 + 3x + 4$
Residu = 2

Exercici 4

a) $(5x + 2)^2 = 25x^2 + 20x + 4$
b) $(3x - 7)^2 = 9x^2 - 42x + 49$
c) $\left(x - \dfrac{1}{2}\right)\left(x + \dfrac{1}{2}\right) = x^2 - \dfrac{1}{4}$
d) $(6x - 2)(6x + 2) = 36x^2 - 4$
e) $(8x + 2)^2 = 64x^2 + 32x + 4$

f) $(6x - 3)^2 = 36x^2 - 36x + 9$

g) $(5x - 4)(5x + 4) = 25x^2 - 16$

h) $(7x + 8)^2 = 49x^2 + 112x + 64$

i) $(5x - 1)^2 = 25x^2 - 10x + 1$

Exercici 5

a) $(x + 1)^2 (3x^2 + 2) - 2(x^4 - x^3 + 2x^2 - 1) = (x^2 + 2x + 1)(3x^2 + 2) - 2(x^4 - x^3 + 2x^2 - 1) =$
$= 3x^4 + 2x^2 + 6x^3 + 4x + 3x^2 + 2 - 2x^4 + 2x^3 - 4x^2 + 2 = x^4 + 8x^3 + x^2 + 4x + 4$

b) $(2x - 3)^2 - (2x^2 + 4x + 1)(x - 2) = (4x^2 - 12x + 9) - (2x^3 + 4x^2 + x - 4x^2 - 8x - 2) =$
$= 4x^2 - 12x + 9 - (2x^3 - 7x - 2) = 4x^2 - 12x + 9 - 2x^3 + 7x + 2 = -2x^3 + 4x^2 - 5x + 11$

c) $\left(\dfrac{2}{3}x - 1\right)(3x + 6) + (x + 1)(x - 1) - (x + 2)^2 = (2x^2 + 4x - 3x - 6) + (x^2 - 1) - (x^2 + 4x + 4) =$
$= 2x^2 + x - 6 + x^2 - 1 - x^2 - 4x - 4 = 2x^2 - 3x - 11$

d) $(x - 1)(x^2 + x)^2 - (x^5 - 5x^4 + x^3 - x^2) = (x - 1)(x^4 + 2x^3 + x^2) - (x^5 - 5x^4 + x^3 - x^2) =$
$= x^5 + 2x^4 + x^3 - x^4 - 2x^3 - x^2 - x^5 + 5x^4 - x^3 + x^2 = 6x^4 - 2x^3$

e) $(x^2 - 2x + 3)(2x + 1) - (4x - 1)^2 = (2x^3 + x^2 - 4x^2 - 2x + 6x + 3) - (16x^2 - 8x + 1) =$
$= 2x^3 - 3x^2 + 4x + 3 - 16x^2 + 8x - 1 = 2x^3 - 19x^2 + 12x + 2$

Exercici 6

a) $5x^5 - 3x^3 + 6x = x(5x^4 - 3x^2 + 6)$

b) $12x^5 - 8x^3 + 4x^2 = 4x^2(3x^3 - 2x + 1)$

c) $x^7 - 3x^6 + 9x^5 = x^5(x^2 - 3x + 9)$

d) $12x^3 - 6x = 6x(2x^2 - 1)$

Exercici 7

a) $25x^2 + 10x + 1 = (5x + 1)^2$

b) $9x^2 - 16 = (3x + 4)(3x - 4)$

c) $16x^2 - 40x + 25 = (4x - 5)^2$

d) $9x^2 - 64 = (3x + 8)(3x - 8)$

e) $9x^2 - 24x + 16 = (3x - 4)^2$

f) $64x^2 - 25 = (8x + 5)(8x - 5)$

Exercici 8

a) $4x^4 - 4x^3 + x^2 = x^2(4x^2 - 4x + 1) = x^2(2x - 1)^2$

b) $2x^3 - 18x = 2x(x^2 - 9) = 2x(x + 3)(x - 3)$

c) $x^6 - 6x^5 + 9x^4 = x^4(x^2 - 6x + 9) = x^4(x - 3)^2$

d) $12x^3 - 75x = 3x(4x^2 - 25) = 3x(2x + 5)(2x - 5)$

e) $9x^5 + 6x^4 + x^3 = x^3(9x^2 + 6x + 1) = x^3(3x - 1)^2$

f) $18x^3 - 2x = 2x(9x^2 - 1) = 2x(3x + 1)(3x - 1)$